

# Multiple Access Wire-tap Channel with Common Message

Hassan Zivari-Fard<sup>†,††</sup>, Bahareh Akhbari<sup>††</sup>, Mahmoud Ahmadian-Attari<sup>††</sup>, Mohammad Reza Aref<sup>†</sup>

<sup>†</sup>Information Systems and Security Lab (ISSL), Sharif University of Technology, Tehran, Iran

Email: hassan\_zivari@ee.kntu.ac.ir, aref@sharif.edu

<sup>††</sup>Department of ECE, K. N. Toosi University of Technology, Tehran, Iran

Email: {akhbari, mahmoud}@eetd.kntu.ac.ir

**Abstract**—In this paper, we study the problem of secret communication over a Multiple Access Wire-tap Channel with Common Message. In this channel, we assume that two transmitters have confidential messages which must be kept secret from the eavesdropper (receiver 2), and both of them have a common message which can be decoded by two receivers. For this setting, in the discrete memoryless case we derive general inner and outer bounds on the secrecy capacity region. Also, we derive inner and outer bounds on the secrecy capacity region for the Gaussian case. Providing numerical examples for the Gaussian case, we illustrate the comparison between derived achievable rate regions, outer bounds and the capacity region of compound multiple access channel.

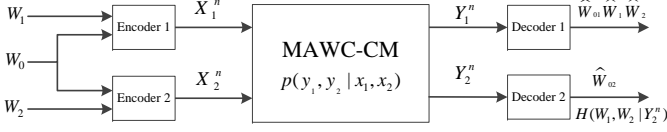
## I. INTRODUCTION

In a seminal paper, the wire-tap channel was introduced by Wyner [1], where a sender wishes to communicate a message to a receiver, while keeping the message secret from an eavesdropper. He established the secrecy capacity for a single-user degraded wire-tap channel. Later, Csiszár and Körner extended the wire-tap channel to a more generalized model called the broadcast channel with confidential messages [2] and found its secrecy capacity.

The problem of secret communication over multi-user channels has received considerable attention, recently [3]–[10] and [11]. In [3], [4], Multiple Access Channel (MAC) with generalized feedback has been considered, where in [3] the encoders do not have to keep their messages secret from each other but their messages should be kept secret from an external eavesdropper, while in [4] each user views the other user as an eavesdropper, and wishes to keep its confidential information as secret as possible from the other user. In [5], a multi-letter achievable rate region and multi-letter outer bound

for the Multiple Access Wire-tap Channel (MAWC) without common message have been derived. The MAWC under the strong secrecy criterion has studied in [6]. In [7], the authors have studied MAWC assuming that there exists a common message and that the eavesdropper is *unable* to decode it. They also have derived a rate region under the strong secrecy criterion. A Gaussian MAWC, for the case where the eavesdropper gets a degraded version of the received signal by legal user, has been considered in [9] and achievable rate region have been found. In [10] general Gaussian MAWC has been considered and an achievable rate region has been derived. The discrete memoryless compound MAC, Gaussian compound MAC with a common message and conferencing decoders, and also the compound MAC when both encoders and decoders cooperate via conferencing links (without any secrecy constraint) have been studied in [12]. In a recent paper [13], we have considered compound MAC with confidential messages in which one of the transmitters' private message ( $W_1$ ) is confidential and only decoded by the first receiver, and kept secret from the second receiver, while the common message ( $W_0$ ) and private message ( $W_2$ ) are decoded by both receivers.

In this paper, we consider Multiple Access Wire-tap Channel with Common Message (MAWC-CM). Actually, in wireless networks, there may be a scenario in which the users may have a *common message* which can be decoded by *all of the users* in addition to the confidential information that wish to be kept secret from illegal users. Motivated by this scenario, we consider MAWC-CM as a building block of this setting. In this model, each transmitter sends its own private message, while both of them have a common message. Both of the transmitters'



**Fig. 1** Multiple Access Wire-tap Channel with Common Message

private messages ( $W_1$  and  $W_2$ ) are confidential and only decoded by the first receiver and kept secret from the second receiver. The common message  $W_0$  is decoded by both receivers (see Fig. 1). For this model we derive *single-letter* inner and outer bounds on the secrecy capacity region. We also consider Gaussian MAWC-CM and derive inner and outer bounds on its secrecy capacity region. Providing some numerical examples for Gaussian MAWC-CM, we compare our derived achievable rate region and outer bound for the Gaussian case with each other and also with the capacity region of Gaussian compound MAC.

This paper is organized as follows. In Section II, the system model is described. In Section III, an outer and inner bounds on the secrecy capacity region of discrete memoryless MAWC-CM are established. An achievable secrecy rate region and an outer bound on the secrecy capacity region of Gaussian MAWC-CM are derived in Section IV.

## II. SYSTEM MODEL

Throughout this paper, the random variables are denoted by capital letters e.g.,  $X$ ,  $Y$ , and their realizations by lower case letters e.g.,  $x$ ,  $y$ . The set of  $\varepsilon$ -strongly jointly typical sequences of length  $n$ , on joint distribution  $p(x, y)$  is denoted by  $A_\varepsilon^n(P_{X,Y})$ . We use  $X_i^n$ , to indicate vector  $(X_{i,1}, X_{i,2}, \dots, X_{i,n})$ , and  $X_{i,j}^k$  to indicate vector  $(X_{i,j}, X_{i,j+1}, \dots, X_{i,k})$ .

Now, consider a discrete memoryless MAWC-CM in Fig. 1 which is denoted by  $(\mathcal{X}_1, \mathcal{X}_2, p(y_1, y_2 | x_1, x_2), \mathcal{Y}_1, \mathcal{Y}_2)$  where  $x_1 \in \mathcal{X}_1$  and  $x_2 \in \mathcal{X}_2$  are the inputs of transmitters, respectively. Also,  $y_1 \in \mathcal{Y}_1$  and  $y_2 \in \mathcal{Y}_2$  are the channel outputs at receiver 1 and receiver 2, respectively. We first define a code for the channel as follows:

**Definition 1:** A  $(M_0, M_1, M_2, n, P_e^n)$  code for the MAWC-CM (Fig. 1) consists of the following: i) Two message sets  $(W_0, W_1)$  and  $(W_0, W_2)$  that are uniformly distributed over  $[1 : M_0] \times [1 : M_1]$  and  $[1 : M_0] \times [1 : M_2]$ , respectively, where messages  $W_u \in \mathcal{W}_u = \{1, 2, \dots, M_u\}$  and  $u = 0, 1, 2$ . ii) A stochastic encoder  $f_k$ ,  $k = 1, 2$  for

transmitter  $k$  is specified by the matrix of conditional probability  $f_k(X_k^n | w_0, w_k)$ , where  $X_k^n \in \mathcal{X}_k^n$ ,  $w_0 \in \mathcal{W}_0$ ,  $w_k \in \mathcal{W}_k$ ,  $k = 1, 2$  are channel inputs, common and private message sets respectively, and  $\sum_{X_k^n} f_k(X_k^n | w_0, w_k) = 1$ ,  $k = 1, 2$ . Note that  $f_k(X_k^n | w_0, w_k)$ ,  $k = 1, 2$  is the probability of encoding message pair  $(w_0, w_k)$ ,  $k = 1, 2$  to the channel input  $X_k^n$ ,  $k = 1, 2$ . iii) The decoding functions are also given by the mappings  $\phi : \mathcal{Y}_1^n \rightarrow \mathcal{W}_0 \times \mathcal{W}_1 \times \mathcal{W}_2$  and  $\rho : \mathcal{Y}_2^n \rightarrow \mathcal{W}_0$ . The first one is at the legitimate receiver and assigns an estimate  $(\hat{W}_{01}, \hat{W}_1, \hat{W}_2) \in [1 : M_0] \times [1 : M_1] \times [1 : M_2]$ , to each received sequence  $y_1^n$ . The second decoder assigns an estimate  $\hat{W}_{02} \in [1 : M_0]$  to each received sequence  $y_2^n$ . The probability of error is defined as,

$$P_e^n = \Pr(\hat{W}_{0j} \neq W_0 \text{ for } j = 1, 2 \text{ or } (\hat{W}_1, \hat{W}_2) \neq (W_1, W_2)). \quad (1)$$

The level of ignorance of eavesdropper with respect to the confidential message is measured by the normalized equivocation  $(1/n)H(W_1, W_2 | Y_2^n)$ .

**Definition 2:** The rate tuple  $(R_0, R_1, R_2)$  is said to be achievable for MAWC-CM, if for any  $\delta > 0$  there exists a  $(M_0, M_1, M_2, n, P_e^n)$  code as

$$P_e^n < \varepsilon \quad (2)$$

$$M_0 \geq 2^{nR_0}, M_1 \geq 2^{nR_1}, M_2 \geq 2^{nR_2} \quad (3)$$

$$R_1 + R_2 - \frac{1}{n}H(W_1, W_2 | Y_2^n) \leq \delta \quad (4)$$

for all sufficiently large  $n$ . The secrecy capacity region is the closure of the set of all achievable rate tuples  $(R_0, R_1, R_2)$ . We note that the perfect secrecy condition (4) implies  $R_1 - (1/n)H(W_1 | Y_2^n) \leq \delta$  and  $R_2 - (1/n)H(W_2 | Y_2^n) \leq \delta$ .

## III. MAIN RESULTS

### A. Outer Bound

**Theorem 1 (Outer bound)** The secrecy capacity region for the MAWC-CM is included in the set of rates satisfying

$$R_0 \leq \min\{I(U; Y_1), I(U; Y_2)\} \quad (5)$$

$$R_1 \leq I(V_1; Y_1 | U) - I(V_1; Y_2 | U) \quad (6)$$

$$R_2 \leq I(V_2; Y_1 | U) - I(V_2; Y_2 | U) \quad (7)$$

$$R_1 + R_2 \leq I(V_1, V_2; Y_1 | U) - I(V_1, V_2; Y_2 | U) \quad (8)$$

$$R_0 + R_1 + R_2 \leq I(V_1, V_2; Y_1) - I(V_1, V_2; Y_2 | U) \quad (9)$$

for some joint distribution

$$p(u)p(v_1, v_2|u)p(x_1|v_1)p(x_2|v_2)p(y_1, y_2|x_1, x_2). \quad (10)$$

**Remark 1** If we set  $W_1 = \emptyset$  or  $W_2 = \emptyset$  (i.e., setting  $V_1 = \emptyset$  or  $V_2 = \emptyset$ ) in Theorem 1, the region reduces to the region of the broadcast channel with confidential messages discussed in [2] by Csiszár and Körner.

*Proof:* We next show that any achievable rate tuples satisfies (5)-(9) for some distribution factorized as (10). Consider a code  $(M_0, M_1, M_2, n, P_e^n)$  for the MAWC-CM. Applying Fano's inequality [14] results in

$$H(W_0, W_1, W_2|Y_1^n) \leq n\varepsilon_1 \quad (11)$$

$$H(W_0|Y_2^n) \leq n\varepsilon_2. \quad (12)$$

where  $\varepsilon_i \rightarrow 0$ ,  $i = 1, 2$  as  $P_e^n \rightarrow 0$ . We first derive the bound on  $R_1$ . Note that the perfect secrecy (4) implies that

$$nR_1 - n\delta \leq H(W_1|Y_2^n). \quad (13)$$

Hence, we derive the bound on  $H(W_1|Y_2^n)$  as following:

$$\begin{aligned} H(W_1|Y_2^n) &= H(W_1|Y_2^n, W_0) + I(W_1; W_0|Y_2^n) \\ &= H(W_1|Y_2^n, W_0) + H(W_0|Y_2^n) - H(W_0|Y_2^n, W_1) \\ &\stackrel{(a)}{\leq} H(W_1|Y_2^n, W_0) + n\varepsilon_2 \\ &\stackrel{(b)}{\leq} H(W_1|Y_2^n, W_0) - H(W_1|Y_1^n, W_0) + n(\varepsilon_1 + \varepsilon_2) \\ &= I(W_1; Y_1^n|W_0) - I(W_1; Y_2^n|W_0) + n(\varepsilon_1 + \varepsilon_2) \\ &= \sum_{i=1}^n [I(W_1; Y_{1,i}|W_0, Y_1^{i-1}) \\ &\quad - I(W_1; Y_{2,i}|W_0, Y_{2,i+1}^n)] + n(\varepsilon_1 + \varepsilon_2) \\ &= \sum_{i=1}^n [I(W_1, Y_{2,i+1}^n; Y_{1,i}|W_0, Y_1^{i-1}) \\ &\quad - I(Y_{2,i+1}^n; Y_{1,i}|W_0, W_1, Y_1^{i-1}) \\ &\quad - I(W_1, Y_1^{i-1}; Y_{2,i}|W_0, Y_{2,i+1}^n) \\ &\quad + I(Y_1^{i-1}; Y_{2,i}|W_0, W_1, Y_{2,i+1}^n)] + n(\varepsilon_1 + \varepsilon_2) \\ &\stackrel{(c)}{=} \sum_{i=1}^n [I(W_1, Y_{2,i+1}^n; Y_{1,i}|W_0, Y_1^{i-1}) \\ &\quad - I(W_1, Y_1^{i-1}; Y_{2,i}|W_0, Y_{2,i+1}^n)] + n(\varepsilon_1 + \varepsilon_2) \end{aligned}$$

$$\begin{aligned} &= \sum_{i=1}^n [I(Y_{2,i+1}^n; Y_{1,i}|W_0, Y_1^{i-1}) \\ &\quad + I(W_1; Y_{1,i}|W_0, Y_1^{i-1}, Y_{2,i+1}^n) \\ &\quad - I(Y_1^{i-1}; Y_{2,i}|W_0, Y_{2,i+1}^n) \\ &\quad - I(W_1; Y_{2,i}|W_0, Y_1^{i-1}, Y_{2,i+1}^n)] + n(\varepsilon_1 + \varepsilon_2) \\ &\stackrel{(d)}{=} \sum_{i=1}^n [I(W_1; Y_{1,i}|W_0, Y_1^{i-1}, Y_{2,i+1}^n) \\ &\quad - I(W_1; Y_{2,i}|W_0, Y_1^{i-1}, Y_{2,i+1}^n)] + n\varepsilon' \\ &\stackrel{(e)}{=} \sum_{i=1}^n [I(V_{1,i}; Y_{1,i}|U_i) - I(V_{1,i}; Y_{2,i}|U_i)] + n\varepsilon' \end{aligned} \quad (14)$$

where (a) and (b) result from Fano's inequalities in (11) and (12), and equalities (c) and (d) results from Csiszár's sum lemma [2, Lemma 7], where we have (15)-(17) and setting  $\varepsilon' = \varepsilon_1 + \varepsilon_2$ . The equality (e) results from the definitions of the random variables as (18)-(20).

$$\begin{aligned} &\sum_{i=1}^n I(Y_{2,i+1}^n; Y_{1,i}|W_0, W_1, Y_1^{i-1}) \\ &= \sum_{i=1}^n I(Y_1^{i-1}; Y_{2,i}|W_0, W_1, Y_{2,i+1}^n) \end{aligned} \quad (15)$$

$$\begin{aligned} &\sum_{i=1}^n I(Y_{2,i+1}^n; Y_{1,i}|W_0, Y_1^{i-1}) \\ &= \sum_{i=1}^n I(Y_1^{i-1}; Y_{2,i}|W_0, Y_{2,i+1}^n) \end{aligned} \quad (16)$$

$$\begin{aligned} &\sum_{i=1}^n I(Y_{2,i+1}^n; Y_{1,i}|W_0, W_1, W_2, Y_1^{i-1}) \\ &= \sum_{i=1}^n I(Y_1^{i-1}; Y_{2,i}|W_0, W_1, W_2, Y_{2,i+1}^n) \end{aligned} \quad (17)$$

$$U_i = W_0, Y_1^{i-1}, Y_{2,i+1}^n \quad (18)$$

$$V_{1,i} = (W_1, U_i) \quad (19)$$

$$V_{2,i} = (W_2, U_i). \quad (20)$$

Now, we have

$$\begin{aligned} H(W_1|Y_2^n) &\leq \\ &n \sum_{i=1}^n p(Q=i) [I(V_{1,Q}; Y_{1,Q}|U_Q, Q=i) \\ &\quad - I(V_{1,Q}; Y_{2,Q}|U_Q, Q=i)] + n\varepsilon' \\ &= n [I(V_{1,Q}; Y_{1,Q}|U_Q, Q) \\ &\quad - I(V_{1,Q}; Y_{2,Q}|U_Q, Q)] + n\varepsilon' \end{aligned}$$

$$\stackrel{(a)}{=} n[I(V_1; Y_1|U) - I(V_1; Y_2|U)] + n\varepsilon' \quad (21)$$

where (a) is due to  $V_{1,Q} = V_1$ ,  $V_{2,Q} = V_2$ ,  $Y_{1,Q} = Y_1$ ,  $Y_{2,Q} = Y_2$ ,  $(U_Q, Q) = U$  and  $Q$  has a uniform distribution over  $\{1, 2, \dots, n\}$  outcomes. Bound on  $H(W_2|Y_2^n)$  derive in a same way. So, we have

$$H(W_2|Y_2^n) \leq n[I(V_2; Y_1|U) - I(V_2; Y_2|U)] + n\varepsilon' \quad (22)$$

Now, we derive the bound on  $n(R_1 + R_2)$  as following:

$$\begin{aligned} H(W_1, W_2|Y_2^n) &= H(W_1, W_2|Y_2^n, W_0) \\ &\quad + I(W_1, W_2; W_0|Y_2^n) \\ &\leq H(W_1, W_2|Y_2^n, W_0) + n\varepsilon_2 \\ &\stackrel{(a)}{\leq} H(W_1, W_2|Y_2^n, W_0) \\ &\quad - H(W_1, W_2|Y_1^n, W_0) + n(\varepsilon_1 + \varepsilon_2) \\ &= I(W_1, W_2; Y_1^n|W_0) \\ &\quad - I(W_1, W_2; Y_2^n|W_0) + n(\varepsilon_1 + \varepsilon_2) \\ &= \sum_{i=1}^n [I(W_1, W_2; Y_{1,i}|W_0, Y_1^{i-1}) \\ &\quad - I(W_1, W_2; Y_{2,i}|W_0, Y_{2,i+1}^n)] + n(\varepsilon_1 + \varepsilon_2) \\ &= \sum_{i=1}^n [I(W_1, W_2, Y_{2,i+1}^n; Y_{1,i}|W_0, Y_1^{i-1}) \\ &\quad - I(Y_{2,i+1}^n; Y_{1,i}|W_0, W_1, W_2, Y_1^{i-1}) \\ &\quad - I(W_1, W_2, Y_1^{i-1}; Y_{2,i}|W_0, Y_{2,i+1}^n) \\ &\quad + I(Y_1^{i-1}; Y_{2,i}|W_0, W_1, W_2, Y_{2,i+1}^n)] + n\varepsilon' \\ &\stackrel{(b)}{=} \sum_{i=1}^n [I(W_1, W_2, Y_{2,i+1}^n; Y_{1,i}|W_0, Y_1^{i-1}) \\ &\quad - I(W_1, W_2, Y_1^{i-1}; Y_{2,i}|W_0, Y_{2,i+1}^n)] + n\varepsilon' \\ &= \sum_{i=1}^n [I(Y_{2,i+1}^n; Y_{1,i}|W_0, Y_1^{i-1}) \\ &\quad + I(W_1, W_2; Y_{1,i}|W_0, Y_1^{i-1}, Y_{2,i+1}^n) \\ &\quad - I(Y_1^{i-1}; Y_{2,i}|W_0, Y_{2,i+1}^n) \\ &\quad - I(W_1, W_2; Y_{2,i}|W_0, Y_1^{i-1}, Y_{2,i+1}^n)] + n\varepsilon' \\ &\stackrel{(c)}{=} \sum_{i=1}^n [I(W_1, W_2; Y_{1,i}|W_0, Y_1^{i-1}, Y_{2,i+1}^n) \\ &\quad - I(W_1, W_2; Y_{2,i}|W_0, Y_1^{i-1}, Y_{2,i+1}^n)] + n\varepsilon' \\ &\stackrel{(d)}{=} \sum_{i=1}^n [I(V_{1,i}, V_{2,i}; Y_{1,i}|U_i) \\ &\quad - I(V_{1,i}, V_{2,i}; Y_{2,i}|U_i)] + n\varepsilon' \end{aligned} \quad (23)$$

where (a) results from Fano's inequality in (11) and (b) and (c) results from (17) and (16) respectively (i.e., Csiszár's sum Lemma), and setting  $\varepsilon' = \varepsilon_1 + \varepsilon_2$ . The equality (d) results from the definitions of the random variables as (18)-(20). Now, by applying the same time-sharing strategy as before, we have

$$R_1 + R_2 \leq I(V_1, V_2; Y_1|U) - I(V_1, V_2; Y_2|U) + \varepsilon'. \quad (24)$$

Now, we derive the bound on  $n(R_0 + R_1 + R_2)$  as following:

$$\begin{aligned} H(W_0, W_1, W_2|Y_2^n) &\stackrel{(a)}{\leq} H(W_0, W_1, W_2|Y_2^n) \\ &\quad - H(W_0, W_1, W_2|Y_1^n) + n\varepsilon_1 \\ &= I(W_0, W_1, W_2; Y_1^n) - I(W_0, W_1, W_2; Y_2^n) + n\varepsilon_1 \\ &= \sum_{i=1}^n [I(W_0, W_1, W_2; Y_{1,i}|Y_1^{i-1}) \\ &\quad - I(W_0, W_1, W_2; Y_{2,i}|Y_{2,i+1}^n)] + n\varepsilon_1 \\ &= \sum_{i=1}^n [I(W_0, W_1, W_2, Y_{2,i+1}^n; Y_{1,i}|Y_1^{i-1}) \\ &\quad - I(Y_{2,i+1}^n; Y_{1,i}|W_0, W_1, W_2, Y_1^{i-1}) \\ &\quad - I(W_0, W_1, W_2, Y_1^{i-1}; Y_{2,i}|Y_{2,i+1}^n) \\ &\quad + I(Y_1^{i-1}; Y_{2,i}|W_0, W_1, W_2, Y_{2,i+1}^n)] + n\varepsilon_1 \\ &\stackrel{(b)}{=} \sum_{i=1}^n [I(W_0, W_1, W_2, Y_{2,i+1}^n; Y_{1,i}|Y_1^{i-1}) \\ &\quad - I(W_0, W_1, W_2, Y_1^{i-1}; Y_{2,i}|Y_{2,i+1}^n)] + n\varepsilon_1 \\ &\leq \sum_{i=1}^n [I(W_0, W_1, W_2, Y_{2,i+1}^n, Y_1^{i-1}; Y_{1,i}) \\ &\quad - I(W_1, W_2; Y_{2,i}|W_0, Y_1^{i-1}, Y_{2,i+1}^n)] + n\varepsilon_1 \\ &\stackrel{(c)}{\leq} \sum_{i=1}^n [I(U_i, V_{1,i}, V_{2,i}; Y_{1,i}) \\ &\quad - I(V_{1,i}, V_{2,i}; Y_{2,i}|U_i)] + n\varepsilon_1 \end{aligned} \quad (25)$$

where (a) results from Fano's inequality in (11), inequality (b) results from (17) and (c) results from the definitions of the random variables as (18)-(20). Now, we have

$$\begin{aligned} H(W_0, W_1, W_2|Y_2^n) &\leq n \sum_{i=1}^n \frac{1}{n} [I(U_i, V_{1,i}, V_{2,i}; Y_{1,i}) \\ &\quad - I(V_{1,i}, V_{2,i}; Y_{2,i}|U_i)] + n\varepsilon_1 \\ &= n \sum_{i=1}^n p(Q=i) [I(U_Q, V_{1,Q}, V_{2,Q}; Y_{1,Q}|Q=i) \\ &\quad - I(V_{1,Q}, V_{2,Q}; Y_{2,Q}|U_Q, Q=i)] + n\varepsilon_1 \end{aligned}$$

$$\begin{aligned}
&= n[I(U_Q, V_{1,Q}, V_{2,Q}; Y_{1,Q}|Q) \\
&\quad - I(V_{1,Q}, V_{2,Q}; Y_{2,Q}|U_Q, Q)] + n\varepsilon_1 \\
&\stackrel{(a)}{=} n[I(U_Q, Q, V_{1,Q}, V_{2,Q}; Y_{1,Q}) \\
&\quad - I(V_{1,Q}, V_{2,Q}; Y_{2,Q}|U_Q, Q)] + n\varepsilon_1 \\
&= n[I(V_{1,Q}, V_{2,Q}; Y_{1,Q}) + I(U_Q; Y_{1,Q}|V_{1,Q}, V_{2,Q}) \\
&\quad + I(Q; Y_{1,Q}|V_{1,Q}, V_{2,Q}, U_Q) \\
&\quad - I(V_{1,Q}, V_{2,Q}; Y_{2,Q}|U_Q, Q)] + n\varepsilon_1 \\
&= n[I(V_1, V_2; Y_1) - I(V_1, V_2; Y_2|U)] + n\varepsilon_1 \quad (26)
\end{aligned}$$

where (a) result from independence of  $Q$  and  $Y_{1,Q}$  and independence of  $Q$  and  $Y_{2,Q}$ .

Now, we derive the bound on  $R_0$  as following:

$$\begin{aligned}
nR_0 &= H(W_0) = I(W_0; Y_1^n) + H(W_0|Y_1^n) \\
&\leq I(W_0; Y_1^n) + n\varepsilon_1 \\
&= \sum_{i=1}^n I(W_0; Y_{1,i}|Y_1^{i-1}) + n\varepsilon_1 \\
&= \sum_{i=1}^n [I(W_0, Y_1^{i-1}; Y_{1,i}) - I(Y_1^{i-1}; Y_{1,i})] + n\varepsilon_1.
\end{aligned}$$

So, we have

$$\begin{aligned}
nR_0 &\leq \sum_{i=1}^n I(W_0, Y_1^{i-1}; Y_{1,i}) + n\varepsilon_1 \\
&\leq \sum_{i=1}^n [I(W_0, Y_1^{i-1}, Y_{2,i+1}^n; Y_{1,i}) \\
&\quad - I(Y_{2,i+1}^n; Y_{1,i}|W_0, Y_1^{i-1})] + n\varepsilon_1 \\
&\leq \sum_{i=1}^n I(W_0, Y_1^{i-1}, Y_{2,i+1}^n; Y_{1,i}) + n\varepsilon_1 \\
&= \sum_{i=1}^n I(U_i; Y_{1,i}) + n\varepsilon_1. \quad (27)
\end{aligned}$$

Now, by applying the same time-sharing strategy as before, we have

$$R_0 \leq I(U; Y_1) + \varepsilon_1. \quad (28)$$

Similarly

$$R_0 \leq I(U; Y_2) + \varepsilon_2. \quad (29)$$

Therefore

$$R_0 \leq \min\{I(U; Y_1), I(U; Y_2)\}. \quad (30)$$

Considering (13), (21), (22),(24),(26) and (30), the region in (5)-(9) is obtained. This completes the proof. ■

## B. Achievability

**Theorem 2** For a discrete memoryless MAWC-CM, the secrecy rate region  $\mathcal{R}(\pi_I)$  is achievable, where  $\mathcal{R}(\pi_I)$  is the closure of the convex hull of all non-negative  $(R_0, R_1, R_2)$  satisfying

$$\begin{cases} R_0 \leq I(U; Y_2) \\ R_1 \leq I(V_1; Y_1|V_2, U) - I(V_1; Y_2|V_2, U) \\ R_2 \leq I(V_2; Y_1|V_1, U) - I(V_2; Y_2|V_1, U) \\ R_1 + R_2 \leq I(V_1, V_2; Y_1|U) - I(V_1, V_2; Y_2|U) \\ R_0 + R_1 + R_2 \leq I(U, V_1, V_2; Y_1) \\ \quad - I(V_1, V_2; Y_2|U) \end{cases}$$

and  $\pi_I$  denotes the class of joint probability mass functions  $p(u, v_1, v_2, x_1, x_2, y_1, y_2)$  that factor as  $p(u)p(v_1|u)p(v_2|u)p(x_1|v_1)p(x_2|v_2)p(y_1, y_2|x_1, x_2)$ .

**Remark 2** If we convert our model to a multiple access wire-tap channel without common message by setting  $U = \emptyset$ ,  $V_1 = X_1$  and  $V_2 = X_2$  in Theorem 2, the region reduces to the region of the [3] and [6].

**Remark 3** If we convert our model to a broadcast channel with confidential messages by setting  $V_1 = \emptyset$  or  $V_2 = \emptyset$ , our region includes the region discussed by Csiszár and Körner in [2].

*Proof:* Fix  $p(u), p(v_1|u), p(v_2|u), p(x_1|v_1)$  and  $p(x_2|v_2)$ . Let

$$R'_1 + R'_2 = I(V_1, V_2; Y_2|U) - \varepsilon, \quad (31)$$

where  $\varepsilon > 0$  and  $\varepsilon \rightarrow 0$  as  $n \rightarrow \infty$ .

1) *Codebook generation:*

- i) Generate  $2^{nR_0}$  codewords  $u^n$ , each is uniformly drawn from the set  $A_\varepsilon^n(P_U)$  indexing by  $u^n(w_0)$ ,  $w_0 \in \{1, \dots, 2^{nR_0}\}$ .
- ii) For each codeword  $u^n(w_0)$ , generate  $2^{n\tilde{R}_1}$  codewords  $v_1^n$  each is uniformly drawn from the set  $A_\varepsilon^{(n)}(P_{V_1|U})$ , where  $\tilde{R}_1 = R_1 + R'_1$ . Then, randomly bin  $2^{n\tilde{R}_1}$  codewords into  $2^{nR_1}$  bins and label them as  $v_1^n(w_0, w_1, l)$ . Here,  $w_1$  is the bin number and  $l \in \mathcal{L} = \{1, \dots, 2^{nR'_1}\}$  is the index of codewords in the bin number  $w_1$ .

The codebook for user 2 is generated in the same way.

2) *Encoding:* To send the message pair  $(w_0, w_1)$ , the encoder  $f_1$  first randomly chooses index  $l$  corresponding to  $(w_0, w_1)$ , and then generates a codeword  $X_1^n$  at random according to  $\prod_{i=1}^n p(x_{1,i}|v_{1,i})$ . Transmitter 2 uses the same way to encode  $(w_0, w_2)$ .

### 3) Decoding and Probability of error:

#### Decoding:

- Receiver 1 declares that the indices of  $(\hat{w}_{01}, \hat{w}_1, \hat{w}_2)$  has been sent if there is a unique tuple of indices  $(\hat{w}_{01}, \hat{w}_1, \hat{w}_2)$  such that  $(u^n(\hat{w}_{01}), v_1^n(\hat{w}_{01}, \hat{w}_1, l), v_2^n(\hat{w}_{01}, \hat{w}_2, l'), y_1^n) \in A_\varepsilon^n(P_{UV_1V_2Y_1})$ .
- Receiver 2 declares that the index of  $u^n(w_0)$  is  $\hat{w}_{02}$  if there is a unique pair such that  $(u^n(\hat{w}_{02}), y_2^n) \in A_\varepsilon^n(P_{UY_2})$ .

**Probability of error:** Using joint decoding [14] and by setting  $\tilde{R}_1 = R_1 + R'_1$  and  $\tilde{R}_2 = R_2 + R'_2$ , it can be shown that the probability of error goes to zero as  $n \rightarrow \infty$  if we choose:

$$R_0 \leq I(U; Y_2) \quad (32)$$

$$R_1 + R'_1 \leq I(V_1; Y_1|V_2, U) \quad (33)$$

$$R_2 + R'_2 \leq I(V_2; Y_2|V_1, U) \quad (34)$$

$$R_1 + R'_1 + R_2 + R'_2 \leq I(V_1, V_2; Y_1|U) \quad (35)$$

$$R_0 + R_1 + R'_1 + R_2 + R'_2 \leq I(U, V_1, V_2; Y_1) \quad (36)$$

#### 4) Equivocation computation:

$$\begin{aligned} H(W_1, W_2|Y_2^n) &\geq H(W_1, W_2|Y_2^n, U^n) \\ &= H(W_1, W_2, Y_2^n|U^n) - H(Y_2^n|U^n) \\ &= H(W_1, W_2, Y_2^n, V_1^n, V_2^n|U^n) \\ &\quad - H(V_1^n, V_2^n|W_1, W_2, Y_2^n, U^n) - H(Y_2^n|U^n) \\ &= H(W_1, W_2, V_1^n, V_2^n|U^n) \\ &\quad + H(Y_2^n|W_1, W_2, V_1^n, V_2^n, U^n) \\ &\quad - H(V_1^n, V_2^n|W_1, W_2, Y_2^n, U^n) - H(Y_2^n|U^n) \\ &\stackrel{(a)}{\geq} H(V_1^n, V_2^n|U^n) - H(V_1^n, V_2^n|W_1, W_2, Y_2^n, U^n) \\ &\quad + H(Y_2^n|V_1^n, V_2^n, U^n) - H(Y_2^n|U^n) \\ &= H(V_1^n, V_2^n|U^n) - H(V_1^n, V_2^n|W_1, W_2, Y_2^n, U^n) \\ &\quad - I(V_1^n, V_2^n; Y_2^n|U^n) \end{aligned} \quad (37)$$

where (a) results from this fact that  $V_1^n$  and  $V_2^n$  are functions of  $W_1$  and  $W_2$  respectively. The first term in (37) is given by:

$$\begin{aligned} H(V_1^n, V_2^n|U^n) &= n\tilde{R}_1 + n\tilde{R}_2 \\ &= n(R_1 + R'_1 + R_2 + R'_2). \end{aligned} \quad (38)$$

We then show that the second term in (37) can be bounded by  $H(V_1^n, V_2^n|W_1, W_2, Y_2^n, U^n) \leq n\varepsilon_1$ , as

$n \rightarrow \infty$  then  $\varepsilon_1 \rightarrow 0$ . We can show that error probability is less than any  $\varepsilon_1 > 0$  if

$$R'_1 \leq I(V_1; Y_2|V_2, U) \quad (39)$$

$$R'_2 \leq I(V_2; Y_2|V_1, U) \quad (40)$$

$$R'_1 + R'_2 \leq I(V_1, V_2; Y_2|U). \quad (41)$$

for sufficiently large  $n$ . In other words, given message  $(w_1, w_2)$ , receiver 2 can decode  $(V_1, V_2)$  until conditions (39), (40) and (41) are satisfied. Therefore, Fano's inequality implies that  $H(V_1^n, V_2^n|W_1 = w_1, W_2 = w_2, Y_2^n, U^n) \leq n\varepsilon_1$ . Hence,

$$\begin{aligned} H(V_1^n, V_2^n|W_1, W_2, Y_2^n, U^n) &= \\ \sum_{w_1} \sum_{w_2} p(W_1 = w_1)p(W_2 = w_2) &\times H(V_1^n, V_2^n|W_1 = w_1, W_2 = w_2, Y_2^n, U^n) \leq n\varepsilon_1. \end{aligned} \quad (42)$$

The last term in (37) is bounded as:

$$I(V_1^n, V_2^n; Y_2^n|U^n) \leq nI(V_1, V_2; Y_2|U) + n\varepsilon_2, \quad (43)$$

as  $n \rightarrow \infty$  then  $\varepsilon_2 \rightarrow 0$  similar to [1, Lemma 1]. By replacing (38), (42) and (43) in (37) we have:

$$\begin{aligned} H(W_1, W_2|Y_2^n) &\geq n(R_1 + R'_1 + R_2 + R'_2) - n\varepsilon_1 \\ &\quad - nI(V_1, V_2; Y_2|U) - n\varepsilon_2 \\ &= n(R_1 + R_2 + R'_1 + R'_2 - I(V_1, V_2; Y_2|U)) - n\varepsilon \\ &= n(R_1 + R_2) - n\varepsilon \end{aligned} \quad (44)$$

where  $\varepsilon = \varepsilon_1 + \varepsilon_2$ . Finally, by using the Fourier-Motzkin procedure [14] to eliminate  $R'_1$  and  $R'_2$  in (31), (32)-(36) and (39)-(41) we obtain the five inequalities in Theorem 2. This completes the proof of Theorem 2. ■

## IV. GAUSSIAN MAWC-CM

In this section, we consider Gaussian MAWC-CM, and derive inner and outer bounds on the secrecy capacity region of it. Relationships between the inputs and outputs of the channel are given by

$$Y_1 = X_1 + X_2 + N_1 \quad (47)$$

$$Y_2 = X_1 + X_2 + N_2 \quad (48)$$

where  $N_j$ ,  $j = 1, 2$  are Gaussian, zero mean Random Variables (RVs), with variances  $\sigma_j^2$ ,  $j = 1, 2$ , and independent of the RVs  $X_1, X_2$ . We impose the power (or energy) constraints  $\frac{1}{n} \sum_{i=1}^n E[X_{j,i}^2] \leq P_j$ ,  $j = 1, 2$ .

### A. Outer Bound

**Theorem 3 (Outer bound)** *The secrecy capacity region for the Gaussian MAWC-CM is included in union on the set of rates satisfying*

$$\left\{ \begin{array}{l} R_0 \leq \min\left\{ C\left(\frac{(1-\beta_1)P_1+(1-\beta_2)P_2+(1-\beta_1\beta_2)\rho\sqrt{P_1P_2}}{\beta_1P_1+\beta_2P_2+2\beta_1\beta_2\rho\sqrt{P_1P_2}+\sigma_1^2}\right), \right. \\ \left. C\left(\frac{(1-\beta_1)P_1+(1-\beta_2)P_2+(1-\beta_1\beta_2)\rho\sqrt{P_1P_2}}{\beta_1P_1+\beta_2P_2+2\beta_1\beta_2\rho\sqrt{P_1P_2}+\sigma_2^2}\right) \right\} \\ R_1 + R_2 \leq \left[ C\left(\frac{\beta_1P_1+\beta_2P_2+2\beta_1\beta_2\rho\sqrt{P_1P_2}}{\sigma_1^2}\right) - C\left(\frac{\beta_1P_1+\beta_2P_2+2\beta_1\beta_2\rho\sqrt{P_1P_2}}{\sigma_2^2}\right) \right] + \\ R_0 + R_1 + R_2 \leq \left[ C\left(\frac{P_1+P_2+2\rho\sqrt{P_1P_2}}{\sigma_1^2}\right) - C\left(\frac{\beta_1P_1+\beta_2P_2+2\beta_1\beta_2\rho\sqrt{P_1P_2}}{\sigma_2^2}\right) \right] + \end{array} \right. \quad (49)$$

where  $C(x) = (1/2) \log(1+x)$  and the union is taken over all  $0 \leq \beta_1 \leq 1$ ,  $0 \leq \beta_2 \leq 1$  and  $0 \leq \rho \leq 1$ .

*Proof:* All the steps in (23), (25) and (30) involve basic properties of mutual information (the chain rule and positivity) that hold irrespective of the continuous or discrete nature of the channel. Therefore, if a rate tuple  $(R_0, R_1, R_2)$  is achievable for the Gaussian MAWC-CM, it must hold that

$$R_0 \leq \min\left\{ \frac{1}{n} \sum_{i=1}^n I(U_i; Y_{1,i}), \frac{1}{n} \sum_{i=1}^n I(U_i; Y_{2,i}) \right\} \quad (50)$$

$$R_1 + R_2 \leq \frac{1}{n} \sum_{i=1}^n [I(V_{1,i}, V_{2,i}; Y_{1,i}|U_i) - I(V_{1,i}, V_{2,i}; Y_{2,i}|U_i)] \quad (51)$$

$$R_0 + R_1 + R_2 \leq \frac{1}{n} \sum_{i=1}^n [I(U_i, V_{1,i}, V_{2,i}; Y_{1,i}) - I(V_{1,i}, V_{2,i}; Y_{2,i}|U_i)]. \quad (52)$$

It remains to upper bound (50)-(52) with terms that depend on the power constraints  $P_1$  and  $P_2$ . We first assume that  $\sigma_1^2 \leq \sigma_2^2$  so that the eavesdropper's channel is a stochastically degraded with respect to the main channel. We expand  $\frac{1}{n} \sum_{i=1}^n I(U_i; Y_{1,i})$  in terms of the differential entropy as

$$\frac{1}{n} \sum_{i=1}^n I(U_i; Y_{1,i}) = \frac{1}{n} \sum_{i=1}^n h(Y_{1,i}) - \frac{1}{n} \sum_{i=1}^n h(Y_{1,i}|U_i) \quad (53)$$

and we bound each sum separately. Notice that since  $Y_{j,i} = X_{1,i} + X_{2,i} + N_{j,i}$ ,  $j = 1, 2$  and  $X_{j,i}$  for

$j = 1, 2$  is independent of  $N_{1,i}$ . So, we have

$$\text{var}(Y_{1,i}) = E[X_{1,i}^2] + E[X_{2,i}^2] + \sigma_1^2 + 2\lambda_i \quad (54)$$

where  $\lambda_i = E[X_{1,i}X_{2,i}]$  and therefore the differential entropy of  $Y_{1,i}$  is upper bounded by the entropy of a Gaussian random variable with the same variance. Hence,

$$\begin{aligned} \frac{1}{n} \sum_{i=1}^n h(Y_{1,i}) &\stackrel{(a)}{\leq} \frac{1}{n} \sum_{i=1}^n \frac{1}{2} \log(2\pi e(E[X_{1,i}^2] + E[X_{2,i}^2] + \sigma_1^2 + 2\lambda_i)) \\ &\stackrel{(b)}{\leq} \frac{1}{2} \log(2\pi e(\frac{1}{n} \sum_{i=1}^n E[X_{1,i}^2] + \frac{1}{n} \sum_{i=1}^n E[X_{2,i}^2] \\ &\quad + \frac{2}{n} \sum_{i=1}^n \lambda_i + \sigma_1^2)), \end{aligned} \quad (55)$$

where (a) is due to  $x \mapsto \log(2\pi ex)$  is a concave function of  $x$  as using Jensen's inequality [14]. Also, (b) results by setting  $Q_1 \triangleq \frac{1}{n} \sum_{i=1}^n E[X_{1,i}^2]$ ,  $Q_2 \triangleq \frac{1}{n} \sum_{i=1}^n E[X_{2,i}^2]$ ,  $Q_3 \triangleq \frac{1}{n} \sum_{i=1}^n \lambda_i$  and  $\rho = \frac{Q_3}{\sqrt{Q_1 Q_2}}$  we finally obtain

$$\frac{1}{n} \sum_{i=1}^n h(Y_{1,i}) \leq \frac{1}{2} \log(2\pi e(Q_1 + Q_2 + 2\rho\sqrt{Q_1 Q_2} + \sigma_1^2)). \quad (56)$$

To bound the second sum  $\frac{1}{n} \sum_{i=1}^n h(Y_i|U_i)$ , notice that

$$\begin{aligned} \frac{1}{n} \sum_{i=1}^n h(Y_{1,i}|U_i) &\leq \frac{1}{n} \sum_{i=1}^n h(Y_{1,i}) \\ &\leq \frac{1}{2} \log(2\pi e(Q_1 + Q_2 + 2\rho\sqrt{Q_1 Q_2} + \sigma_1^2)). \end{aligned} \quad (57)$$

Moreover, because  $U_i \rightarrow (V_{1,i}, V_{2,i}) \rightarrow (X_{1,i}, X_{2,i}) \rightarrow (Y_{1,i}, Y_{2,i})$  forms a Markov chain, we have

$$\begin{aligned} \frac{1}{n} \sum_{i=1}^n h(Y_{1,i}|U_i) &\geq \frac{1}{n} \sum_{i=1}^n h(Y_{1,i}|U_i, X_{1,i}, X_{2,i}) \\ &= \frac{1}{n} \sum_{i=1}^n h(Y_{1,i}|X_{1,i}, X_{2,i}) \\ &= \frac{1}{2} \log(2\pi e\sigma_1^2). \end{aligned} \quad (58)$$

Since  $x, y \mapsto \frac{1}{2} \log(2\pi e(xQ_1 + yQ_2 + 2xy\rho\sqrt{Q_1 Q_2} + \sigma_1^2))$  is continuous function on interval,  $x \in [0, 1]$

and  $y \in [0, 1]$ , a two-dimensional intermediate-value theorem ensures the existence of  $\beta_1, \beta_2 \in [0, 1]$  such that

$$\frac{1}{n} \sum_{i=1}^n h(Y_{1,i}|U_i) = \frac{1}{2} \log(2\pi e(\beta_1 Q_1 + \beta_2 Q_2 + 2\beta_1 \beta_2 \rho \sqrt{Q_1 Q_2} + \sigma_1^2)). \quad (59)$$

By substituting (56) and (59) into (53), we obtain

$$\begin{aligned} \frac{1}{n} \sum_{i=1}^n I(U_i; Y_{1,i}) &\leq \frac{1}{2} \log(2\pi e(Q_1 + Q_2 + 2\rho \sqrt{Q_1 Q_2} + \sigma_1^2)) - \\ &\frac{1}{2} \log(2\pi e(\beta_1 Q_1 + \beta_2 Q_2 + 2\beta_1 \beta_2 \rho \sqrt{Q_1 Q_2} + \sigma_1^2)) = \\ &\frac{1}{2} \log(1 + \frac{(1-\beta_1)Q_1 + (1-\beta_2)Q_2 + 2(1-\beta_1\beta_2)\rho\sqrt{Q_1 Q_2}}{\beta_1 Q_1 + \beta_2 Q_2 + 2\beta_1 \beta_2 \rho \sqrt{Q_1 Q_2} + \sigma_1^2}). \end{aligned} \quad (60)$$

Now, we need to upper bound  $\frac{1}{n} \sum_{i=1}^n I(U_i; Y_{2,i})$ . Note that we can repeat the steps leading to (56) with  $Y_{2,i}$  in place of  $Y_{1,i}$  to obtain

$$\frac{1}{n} \sum_{i=1}^n h(Y_{2,i}) \leq \frac{1}{2} \log(2\pi e(Q_1 + Q_2 + 2\rho \sqrt{Q_1 Q_2} + \sigma_2^2)). \quad (61)$$

So, we need to derive a lower bound for  $\frac{1}{n} \sum_{i=1}^n h(Y_{2,i}|U_i)$  as a function of  $Q_1, Q_2, \beta_2, \beta_1$  and  $\rho$ . Since we have assume that the eavesdropper's channel is stochastically degraded with respect to the main channel, we can write  $Y_{2,i} = Y_{1,i} + N'_i$  with  $N'_i \sim \mathcal{N}(0, \sigma_2^2 - \sigma_1^2)$ . Applying the Entropy Power Inequality (EPI) [14] to the RV  $Y_{2,i}$  conditioned on  $U_i = u_i$ , we have

$$\begin{aligned} h(Y_{2,i}|U_i = u_i) &= h(Y_{1,i} + N'_i|U_i = u_i) \\ &\geq \frac{1}{2} \log(2^{2h(Y_{1,i}|U_i=u_i)} + 2^{2h(N'_i|U_i=u_i)}) \\ &= \frac{1}{2} \log(2^{2h(Y_{1,i}|U_i=u_i)} + 2\pi e(\sigma_2^2 - \sigma_1^2)). \end{aligned} \quad (62)$$

Hence,

$$\begin{aligned} \frac{1}{n} \sum_{i=1}^n h(Y_{2,i}|U_i) &= \frac{1}{n} \sum_{i=1}^n E_{U_i}[h(Y_{2,i}|U_i)] \\ &\stackrel{(a)}{\geq} \frac{1}{2n} \sum_{i=1}^n E_{U_i}[\log(2^{2h(Y_{1,i}|U_i)} + 2\pi e(\sigma_2^2 - \sigma_1^2))] \\ &\stackrel{(b)}{\geq} \frac{1}{2n} \sum_{i=1}^n \log(2^{2E_{U_i}[h(Y_{1,i}|U_i)]} + 2\pi e(\sigma_2^2 - \sigma_1^2)) \\ &= \frac{1}{2n} \sum_{i=1}^n \log(2^{2h(Y_{1,i}|U_i)} + 2\pi e(\sigma_2^2 - \sigma_1^2)) \end{aligned}$$

$$\begin{aligned} &\stackrel{(c)}{\geq} \frac{1}{2} \log(2^{\frac{2}{n} \sum_{i=1}^n h(Y_{1,i}|U_i)} + 2\pi e(\sigma_2^2 - \sigma_1^2)) \\ &\stackrel{(d)}{=} \frac{1}{2} \log(2\pi e(\beta_1 Q_1 + \beta_2 Q_2 + 2\beta_1 \beta_2 \rho \sqrt{Q_1 Q_2} + \sigma_1^2 + 2\pi e(\sigma_2^2 - \sigma_1^2)) \\ &= \frac{1}{2} \log(2\pi e(\beta_1 Q_1 + \beta_2 Q_2 + 2\beta_1 \beta_2 \rho \sqrt{Q_1 Q_2} + \sigma_2^2)), \end{aligned} \quad (63)$$

where (a) follows from EPI. Both (b) and (c) follow from the convexity of the function  $x \mapsto \log(2^x + c)$  for  $c \in \mathbb{R}_+$  and Jensen's inequality, while (d) follows from (59). Hence,

$$\begin{aligned} \frac{1}{n} \sum_{i=1}^n I(U_i; Y_{2,i}) &= \frac{1}{n} \sum_{i=1}^n h(Y_{2,i}) - h(Y_{2,i}|U_i) \leq \\ &\leq \frac{1}{2} \log(2\pi e(Q_1 + Q_2 + 2\rho \sqrt{Q_1 Q_2} + \sigma_2^2)) \\ &\quad - \frac{1}{2} \log(2\pi e(\beta_1 Q_1 + \beta_2 Q_2 + 2\beta_1 \beta_2 \rho \sqrt{Q_1 Q_2} + \sigma_2^2)) \\ &= \frac{1}{2} \log(1 + \frac{(1-\beta_1)Q_1 + (1-\beta_2)Q_2 + 2(1-\beta_1\beta_2)\rho\sqrt{Q_1 Q_2}}{\beta_1 Q_1 + \beta_2 Q_2 + 2\beta_1 \beta_2 \rho \sqrt{Q_1 Q_2} + \sigma_2^2}). \end{aligned} \quad (64)$$

where the inequality follows from (61) and (63). By substituting (60) and (64) into (50), we obtain

$$R_0 \leq \min \left( C \left( \frac{(1-\beta_1)Q_1 + (1-\beta_2)Q_2 + 2(1-\beta_1\beta_2)\rho\sqrt{P_1 P_2}}{\beta_1 Q_1 + \beta_2 Q_2 + 2\beta_1 \beta_2 \rho \sqrt{P_1 P_2} + \sigma_1^2} \right), C \left( \frac{(1-\beta_1)Q_1 + (1-\beta_2)Q_2 + 2(1-\beta_1\beta_2)\rho\sqrt{P_1 P_2}}{\beta_1 Q_1 + \beta_2 Q_2 + 2\beta_1 \beta_2 \rho \sqrt{P_1 P_2} + \sigma_2^2} \right) \right). \quad (65)$$

Now, we derive the bound on  $R_1 + R_2$ ,

$$\begin{aligned} \frac{1}{n} \sum_{i=1}^n [I(V_{1,i}, V_{2,i}; Y_{1,i}|U_i) - I(V_{1,i}, V_{2,i}; Y_{2,i}|U_i)] \\ &= \frac{1}{n} \sum_{i=1}^n [I(V_{1,i}, V_{2,i}, X_{1,i}, X_{2,i}; Y_{1,i}|U_i) \\ &\quad - I(X_{1,i}, X_{2,i}; Y_{1,i}|U_i, V_{1,i}, V_{2,i}) \\ &\quad - I(V_{1,i}, V_{2,i}, X_{1,i}, X_{2,i}; Y_{2,i}|U_i) \\ &\quad + I(X_{1,i}, X_{2,i}; Y_{2,i}|U_i, V_{1,i}, V_{2,i})] \\ &= \frac{1}{n} \sum_{i=1}^n [I(X_{1,i}, X_{2,i}; Y_{1,i}|U_i) \\ &\quad + I(V_{1,i}, V_{2,i}; Y_{1,i}|U_i, X_{1,i}, X_{2,i}) \\ &\quad - I(X_{1,i}, X_{2,i}; Y_{1,i}|U_i, V_{1,i}, V_{2,i}) \\ &\quad - I(X_{1,i}, X_{2,i}; Y_{2,i}|U_i) \\ &\quad - I(V_{1,i}, V_{2,i}; Y_{2,i}|U_i, X_{1,i}, X_{2,i}) \\ &\quad + I(X_{1,i}, X_{2,i}; Y_{2,i}|U_i, V_{1,i}, V_{2,i})] \end{aligned}$$



$$\begin{aligned}
& \stackrel{(a')}{=} \frac{1}{n} \sum_{i=1}^n [I(X_{1,i}, X_{2,i}; Y_{1,i} | U_i) \\
& \quad - I(X_{1,i}, X_{2,i}; Y_{1,i} | U_i, V_{1,i}, V_{2,i}) \\
& \quad - I(X_{1,i}, X_{2,i}; Y_{2,i} | U_i) \\
& \quad + I(X_{1,i}, X_{2,i}; Y_{2,i} | U_i, V_{1,i}, V_{2,i})] \\
& = \frac{1}{n} \sum_{i=1}^n [I(X_{1,i}, X_{2,i}; Y_{1,i} | U_i) \\
& \quad - I(X_{1,i}, X_{2,i}; Y_{1,i}, Y_{2,i} | U_i, V_{1,i}, V_{2,i}) \\
& \quad + I(X_{1,i}, X_{2,i}; Y_{2,i} | U_i, V_{1,i}, V_{2,i}, Y_{1,i}) \\
& \quad - I(X_{1,i}, X_{2,i}; Y_{2,i} | U_i) \\
& \quad + I(X_{1,i}, X_{2,i}; Y_{2,i} | U_i, V_{1,i}, V_{2,i})] \\
& \stackrel{(a'')}{=} \frac{1}{n} \sum_{i=1}^n [I(X_{1,i}, X_{2,i}; Y_{1,i} | U_i) \\
& \quad - I(X_{1,i}, X_{2,i}; Y_{1,i}, Y_{2,i} | U_i, V_{1,i}, V_{2,i}) \\
& \quad - I(X_{1,i}, X_{2,i}; Y_{2,i} | U_i) \\
& \quad + I(X_{1,i}, X_{2,i}; Y_{2,i} | U_i, V_{1,i}, V_{2,i})] \\
& \stackrel{(a)}{\leq} \frac{1}{n} \sum_{i=1}^n [I(X_{1,i}, X_{2,i}; Y_{1,i} | U_i) \\
& \quad - I(X_{1,i}, X_{2,i}; Y_{2,i} | U_i)] \tag{66}
\end{aligned}$$

where (a') follows from  $I(V_{1,i}, V_{2,i}; Y_{1,i} | U_i, X_{1,i}, X_{2,i}) = 0$  since  $U_i \rightarrow (V_{1,i}, V_{2,i}) \rightarrow (X_{1,i}, X_{2,i}) \rightarrow (Y_{1,i}, Y_{2,i})$  forms a Markov chain, (a'') follows from  $I(X_{1,i}, X_{2,i}; Y_{2,i} | U_i, V_{1,i}, V_{2,i}, Y_{1,i}) = 0$  since  $Y_{2,i}$  is stochastically degraded with respect to  $Y_{1,i}$ , and (a) follows from  $I(X_{1,i}, X_{2,i}; Y_{2,i} | U_i, V_{1,i}, V_{2,i}) \leq I(X_{1,i}, X_{2,i}; Y_{1,i}, Y_{2,i} | U_i, V_{1,i}, V_{2,i})$ . Next we have

$$\begin{aligned}
& \frac{1}{n} \sum_{i=1}^n [I(X_{1,i}, X_{2,i}; Y_{1,i} | U_i) - I(X_{1,i}, X_{2,i}; Y_{2,i} | U_i)] \\
& = \frac{1}{n} \sum_{i=1}^n [h(Y_{1,i} | U_i) - h(Y_{1,i} | U_i, X_{1,i}, X_{2,i}) \\
& \quad - h(Y_{2,i} | U_i) + h(Y_{2,i} | U_i, X_{1,i}, X_{2,i})] \\
& \stackrel{(a)}{\leq} \frac{1}{2} \log \left( 1 + \frac{\beta_1 Q_1 + \beta_2 Q_2 + 2\beta_1 \beta_2 \rho \sqrt{Q_1 Q_2}}{\sigma_1^2} \right) \\
& \quad - \frac{1}{2} \log \left( 1 + \frac{\beta_1 Q_1 + \beta_2 Q_2 + 2\beta_1 \beta_2 \rho \sqrt{Q_1 Q_2}}{\sigma_2^2} \right) \tag{67}
\end{aligned}$$

where (a) results from (58), (59), (63) and  $h(Y_{2,i} | U_i, X_{1,i}, X_{2,i}) = \frac{1}{2} \log(2\pi e \sigma_2^2)$ .

Now we derive the bound on  $R_0 + R_1 + R_2$ ,

$$\begin{aligned}
& \frac{1}{n} \sum_{i=1}^n [I(U_i, V_{1,i}, V_{2,i}; Y_{1,i}) - I(V_{1,i}, V_{2,i}; Y_{2,i} | U_i)] \\
& = \frac{1}{n} \sum_{i=1}^n [I(U_i, V_{1,i}, V_{2,i}, X_{1,i}, X_{2,i}; Y_{1,i}) \\
& \quad - I(X_{1,i}, X_{2,i}; Y_{1,i} | U_i, V_{1,i}, V_{2,i}) \\
& \quad - I(V_{1,i}, V_{2,i}, X_{1,i}, X_{2,i}; Y_{2,i} | U_i) \\
& \quad + I(X_{1,i}, X_{2,i}; Y_{2,i} | U_i, V_{1,i}, V_{2,i})] \\
& = \frac{1}{n} \sum_{i=1}^n [I(U_i, X_{1,i}, X_{2,i}; Y_{1,i}) \\
& \quad + I(V_{1,i}, V_{2,i}; Y_{1,i} | U_i, X_{1,i}, X_{2,i}) \\
& \quad - I(X_{1,i}, X_{2,i}; Y_{1,i} | U_i, V_{1,i}, V_{2,i}) \\
& \quad - I(X_{1,i}, X_{2,i}; Y_{2,i} | U_i) \\
& \quad - I(V_{1,i}, V_{2,i}; Y_{2,i} | U_i, X_{1,i}, X_{2,i}) \\
& \quad + I(X_{1,i}, X_{2,i}; Y_{2,i} | U_i, V_{1,i}, V_{2,i})] \\
& \stackrel{(a)}{=} \frac{1}{n} \sum_{i=1}^n [I(U_i, X_{1,i}, X_{2,i}; Y_{1,i}) \\
& \quad - I(X_{1,i}, X_{2,i}; Y_{1,i} | U_i, V_{1,i}, V_{2,i}) \\
& \quad - I(X_{1,i}, X_{2,i}; Y_{2,i} | U_i) \\
& \quad + I(X_{1,i}, X_{2,i}; Y_{2,i} | U_i, V_{1,i}, V_{2,i})] \\
& = \frac{1}{n} \sum_{i=1}^n [I(U_i, X_{1,i}, X_{2,i}; Y_{1,i}) \\
& \quad - I(X_{1,i}, X_{2,i}; Y_{1,i}, Y_{2,i} | U_i, V_{1,i}, V_{2,i}) \\
& \quad + I(X_{1,i}, X_{2,i}; Y_{2,i} | U_i, V_{1,i}, V_{2,i}, Y_{1,i}) \\
& \quad - I(X_{1,i}, X_{2,i}; Y_{2,i} | U_i) \\
& \quad + I(X_{1,i}, X_{2,i}; Y_{2,i} | U_i, V_{1,i}, V_{2,i})] \\
& \stackrel{(b)}{=} \frac{1}{n} \sum_{i=1}^n [I(U_i, X_{1,i}, X_{2,i}; Y_{1,i}) \\
& \quad - I(X_{1,i}, X_{2,i}; Y_{1,i}, Y_{2,i} | U_i, V_{1,i}, V_{2,i}) \\
& \quad - I(X_{1,i}, X_{2,i}; Y_{2,i} | U_i) \\
& \quad + I(X_{1,i}, X_{2,i}; Y_{2,i} | U_i, V_{1,i}, V_{2,i})] \\
& \stackrel{(c)}{\leq} \frac{1}{n} \sum_{i=1}^n [I(U_i, X_{1,i}, X_{2,i}; Y_{1,i}) \\
& \quad - I(X_{1,i}, X_{2,i}; Y_{2,i} | U_i)] \tag{68}
\end{aligned}$$

where (a) follows from  $I(V_{1,i}, V_{2,i}; Y_{1,i} | U_i, X_{1,i}, X_{2,i}) = 0$  since  $U_i \rightarrow (V_{1,i}, V_{2,i}) \rightarrow (X_{1,i}, X_{2,i}) \rightarrow (Y_{1,i}, Y_{2,i})$  forms a Markov chain, (b) follows from

$I(X_{1,i}, X_{2,i}; Y_{2,i} | U_i, V_{1,i}, V_{2,i}, Y_{1,i}) = 0$  since  $Y_{2,i}$  is stochastically degraded with respect to  $Y_{1,i}$ , and (c) follows from  $I(X_{1,i}, X_{2,i}; Y_{2,i} | U_i, V_{1,i}, V_{2,i}) \leq I(X_{1,i}, X_{2,i}; Y_{1,i}, Y_{2,i} | U_i, V_{1,i}, V_{2,i})$ . Next we have

$$\begin{aligned}
& \frac{1}{n} \sum_{i=1}^n [I(U_i, X_{1,i}, X_{2,i}; Y_{1,i}) - I(X_{1,i}, X_{2,i}; Y_{2,i} | U_i)] \\
&= \frac{1}{n} \sum_{i=1}^n [h(Y_{1,i}) - h(Y_{1,i} | U_i, X_{1,i}, X_{2,i}) \\
&\quad - h(Y_{2,i} | U_i) + h(Y_{2,i} | U_i, X_{1,i}, X_{2,i})] \\
&\stackrel{(b)}{\leq} \frac{1}{2} \log 2\pi e \left(1 + \frac{Q_1 + Q_2 + 2\rho\sqrt{P_1 P_2}}{\sigma_1^2}\right) \\
&\quad - \frac{1}{2} \log 2\pi e \left(1 + \frac{\beta_1 Q_1 + \beta_2 Q_2 + 2\beta_1 \beta_2 \rho\sqrt{P_1 P_2}}{\sigma_2^2}\right)
\end{aligned} \tag{69}$$

where (b) follows from (56), (58), (63) and  $h(Y_{2,i} | U_i, X_{1,i}, X_{2,i}) = \frac{1}{2} \log(2\pi e \sigma_2^2)$ .

If  $\sigma_1^2 \geq \sigma_2^2$ , then the main channel is stochastically degraded with respect to the eavesdropper's channel and  $R_1 = R_2 = 0$  by virtue of [11, Proposition 3.4]. By swapping the roles of  $Y_{1,i}$  and  $Y_{2,i}$  in the proof, the reader can verify that (65) still holds. We combine the two cases  $\sigma_1^2 \leq \sigma_2^2$  and  $\sigma_1^2 \geq \sigma_2^2$  by writing (49). Notice that (65), (67) and (69) are increasing functions of  $Q_1$  and  $Q_2$  hence, by definition  $Q_j = (1/n) \sum_{i=1}^n E[X_{j,i}^2] \leq P_j$ ,  $j = 1, 2$  the inequalities in Theorem 3 are hold. This completes the proof. ■

### B. Inner Bound

**Theorem 4** *An inner bound on the secrecy capacity region of Gaussian MAWC-CM is:*

$$\bigcup \left\{ \begin{aligned} R_0 &\leq C\left(\frac{\beta_1^2 P_1 + \beta_2^2 P_2 + 2\beta_1 \beta_2 \sqrt{P_1 P_2}}{(1-\beta_1^2)P_1 + (1-\beta_2^2)P_2 + \sigma_2^2}\right) \\ R_1 &\leq C\left(\frac{(1-\beta_1^2)P_1}{\sigma_1^2}\right) - C\left(\frac{(1-\beta_1^2)P_1}{\sigma_2^2}\right) \\ R_2 &\leq C\left(\frac{(1-\beta_2^2)P_2}{\sigma_1^2}\right) - C\left(\frac{(1-\beta_2^2)P_2}{\sigma_2^2}\right) \\ R_1 + R_2 &\leq C\left(\frac{(1-\beta_1^2)P_1 + (1-\beta_2^2)P_2}{\sigma_1^2}\right) \\ &\quad - C\left(\frac{(1-\beta_1^2)P_1 + (1-\beta_2^2)P_2}{\sigma_2^2}\right) \\ R_0 + R_1 + R_2 &\leq C\left(\frac{P_1 + P_2 + 2\beta_1 \beta_2 \sqrt{P_1 P_2}}{\sigma_1^2}\right) \\ &\quad - C\left(\frac{(1-\beta_1^2)P_1 + (1-\beta_2^2)P_2}{\sigma_2^2}\right) \end{aligned} \right.$$

where  $C(x) = (1/2) \log(1+x)$  and the union is taken over all  $0 \leq \beta_1 \leq 1$  and  $0 \leq \beta_2 \leq 1$ .

*Proof:* Let  $U$ ,  $K_1$  and  $K_2$  be independent, unit variance, Gaussian RVs, and define  $V_1 = X_1$ ,  $V_2 = X_2$  and:

$$X_1 = \sqrt{P_1} \beta_1 U + \sqrt{P_1(1-\beta_1^2)} K_1 \tag{70}$$

$$X_2 = \sqrt{P_2} \beta_2 U + \sqrt{P_2(1-\beta_2^2)} K_2. \tag{71}$$

So, following from Theorem 2, defining the above RVs as in (47)-(48) and (70)-(71) the region in (70) is derived. ■

In order to compare our derive bounds we dderive the capacity region of the Gaussian compound MAC.

**Theorem 5** *The capacity region of compound Gaussian MAC is given by:*

$$\bigcup \left\{ \begin{aligned} 0 \leq R_1 &\leq \min\left\{C\left(\frac{P_1(1-\beta_1^2)}{\sigma_1^2}\right), C\left(\frac{P_1(1-\beta_1^2)}{\sigma_2^2}\right)\right\} \\ 0 \leq R_2 &\leq \min\left\{C\left(\frac{P_2(1-\beta_2^2)}{\sigma_1^2}\right), C\left(\frac{P_2(1-\beta_2^2)}{\sigma_2^2}\right)\right\} \\ 0 \leq R_1 + R_2 &\leq \min\left\{C\left(\frac{P_1(1-\beta_1^2) + P_2(1-\beta_2^2)}{\sigma_1^2}\right), \right. \\ &\quad \left. C\left(\frac{P_1(1-\beta_1^2) + P_2(1-\beta_2^2)}{\sigma_2^2}\right)\right\} \\ 0 \leq R_0 + R_1 + R_2 &\leq \min\left\{C\left(\frac{P_1 + P_2 + 2\sqrt{P_1 P_2} \beta_1 \beta_2}{\sigma_1^2}\right), \right. \\ &\quad \left. C\left(\frac{P_1 + P_2 + 2\sqrt{P_1 P_2} \beta_1 \beta_2}{\sigma_2^2}\right)\right\} \end{aligned} \right. \tag{72}$$

where  $C(x) = (1/2) \log(1+x)$  and the union is taken over all  $0 \leq \beta_1 \leq 1$  and  $0 \leq \beta_2 \leq 1$ .

*Proof:* Considering Propositions 6.1 and 6.2 in [12] which are outer and inner bounds on the capacity region of compound MAC with conferencing links, then ignoring conferencing links in that model (i.e, setting  $C_{12} = C_{21} = 0$  in [12]), and by adopting it to our defined channel parameters in (47)-(48), the region in (72) is derived. ■

### C. Examples

In this part, we provide numerical examples and compare inner and outer bounds on the secrecy capacity region of Gaussian MAWC-WC. We also compare these bounds with the capacity region of compound MAC illustrated in (72). As an example, for the values  $P_1 = P_2 = 1$ ,  $\sigma_1^2 = .1$  and  $\sigma_2^2 = .3$  the outer bound in Theorem 3 and the achievable rate region in Theorem 4 are depicted in Fig. 2. In order to compare these bounds with the capacity region of compound MAC in Theorem 5, these bounds

and also the capacity region of compound MAC are depicted in Fig. 3 for the same parameters as before.

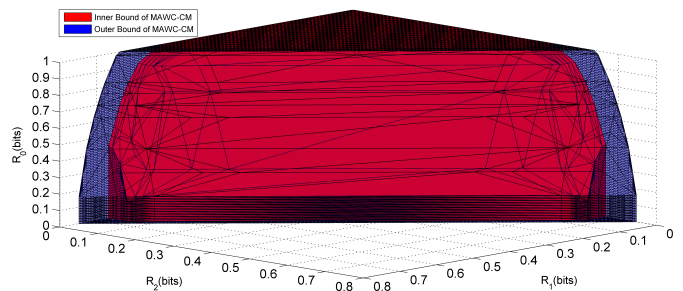
As it can be seen in Fig. 3, for this scenario the achievable rate and outer bound on  $R_0$  (rate of  $W_0$  which is decoded by both receivers) are the same for both MAWC-WC and compound MAC models. Moreover, as it can be seen from Fig. 3, for these channel parameters, the achievable rates and outer bounds on  $R_1$  and  $R_2$  (rate of  $W_1$  and  $W_2$  respectively, which is decoded by the receiver 1) for MAWC-WC is less than that for compound MAC due to secrecy constraint for decoding messages  $W_1$  and  $W_2$ .

According to (72), it is clear that if the noise power of receiver 2 (i.e.,  $\sigma_2^2$ ) increases, the capacity region of compound MAC does not increase (i.e., the capacity region may remain as before or decreases). On the other hand, there exist scenarios for MAWC-WC (refer to (70)) for which increasing  $\sigma_2^2$  results in increasing its achievable rate region. For comparison, assume changing  $\sigma_2^2 = 0.3$  to  $\sigma_2^2 = 0.6$  in the above example. As it can be seen in Fig. 4, the achievable rate region of MAWC-WC is larger than the capacity region of compound MAC for the new parameters. This can be interpreted as follows: transmitted signal of transmitter 1 is extremely attenuated at the receiver 2. So, for this case, not forcing the receiver 2 to decode  $W_1$  and  $W_2$  (keeping  $W_1$  and  $W_2$  as a secret for receiver 2) can increase the achievable rate region.

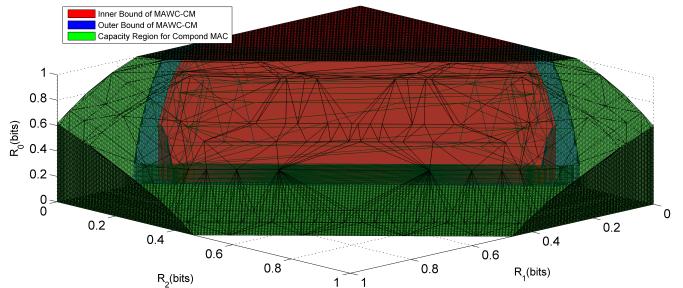
Also, it should be noted that according to (70), for the scenarios where  $\sigma_2^2$  equals or less than  $\sigma_1^2$ , the achievable rates  $R_1$  and  $R_2$  are zero. This happens since for these scenarios the channel gain between transmitter 1 and receiver 2 (i.e., illegal user in terms of messages  $W_1$  and  $W_2$ ) is equal or better than the channel gain between transmitter 1 and receiver 1 (i.e., legitimate user in terms of message  $W_1$  and  $W_2$ ), and this makes zero  $R_1$  and  $R_2$ .

## V. CONCLUSIONS

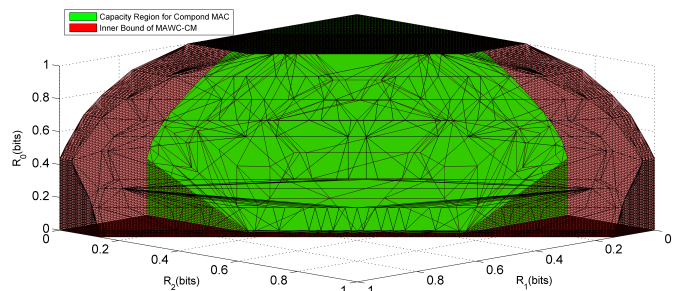
In this paper, we have studied the secrecy capacity region of the Multiple Access Wire-tap Channel with Common Message (MAWC-CM). We have obtained inner and outer bounds on the secrecy capacity for the general MAWC-CM. We have further studied Gaussian MAWC-CM and derived inner and outer bounds on the secrecy capacity region. Providing numerical examples for the Gaussian case, we have shown that there are scenarios for which



**Fig. 2** Achievable rate region and Outer bound of Gaussian MAWC-CM for  $P_1 = P_2 = 1$ ,  $\sigma_1^2 = .1$  and  $\sigma_2^2 = .3$ .



**Fig. 3** Achievable rate region and Outer bound of Gaussian MAWC-CM and the Capacity region of Gaussian compound MAC for  $P_1 = P_2 = 1$ ,  $\sigma_1^2 = .1$  and  $\sigma_2^2 = .3$ .



**Fig. 4** Achievable rate region of Gaussian MAWC-CM and the Capacity region of Gaussian compound MAC for  $P_1 = P_2 = 1$ ,  $\sigma_1^2 = .1$  and  $\sigma_2^2 = .6$ .

keeping secret some of the transmitted messages from the illegal user can increase the achievable rate region.

## REFERENCES

- [1] A. D. Wyner, "The wire-tap channel," *Bell System Technical Journal*, vol. 57, no. 8, pp. 1355–1367, Oct 1975.
- [2] I. Csiszár and J. Körner, "Broadcast channels with confidential messages," *IEEE Trans. Inf. Theory*, vol. 24, no. 3, pp. 339–348, May 1978.
- [3] X. Tang, R. Liu, P. Spasojević, and H. V. Poor, "Multiple access channels with generalized feedback and confidential messages," in *Proc. IEEE Info. Theory Workshop (ITW)*, CA, USA, Sep 2007, pp. 608–613.
- [4] L. Liang and H. V. Poor, "Multiple-access channels with confidential messages," *IEEE Trans. Inf. Theory*, vol. 3, no. 3, pp. 976–1002, Mar 2008.

- [5] E. Ekrem and S. Ulukus, "On the secrecy of multiple access wiretap channel," in *Proc. 46th Annual Allerton Conference on Communication, Control, and Computing*, Monticello, IL, Sep 2008, pp. 1014–1021.
- [6] M. H. Yassaee and M. R. Aref, "Multiple access wiretap channels with strong secrecy," in *Proc. IEEE Info. Theory Workshop (ITW)*, Dublin, Ireland, Sep 2010, pp. 1–5.
- [7] M. Wiese and H. Boche, "An achievable region for the wiretap multiple-access channel with common message," in *Proc. IEEE Int. Symp. on Info. Theory (ISIT)*, Cambridge, MA, Jul 2012, pp. 249–253.
- [8] R. Liu, I. Marić, R. D. Yates, and P. Spasojević, "The discrete memoryless multiple access channel with confidential messages," in *Proc. IEEE Int. Symp. on Info. Theory (ISIT)*, Seattle, WA, Sep 2006, pp. 957–961.
- [9] E. Tekin and A. Yener, "The Gaussian multiple access wire-tap channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 12, pp. 5747–5755, Dec 2008.
- [10] —, "The general Gaussian multiple-access and two-way wiretap channels: Achievable rates and cooperative jamming," *IEEE Trans. Inf. Theory*, vol. 54, no. 6, pp. 2735–2751, Jun 2008.
- [11] M. Bloch and J. Barros, *Physical-Layer Security: From Information Theory to Security Engineering*, 1st ed. Cambridge, U.K: Cambridge University Press, 2011.
- [12] O. Simeone, D. Gündüz, H. V. Poor, A. J. Goldsmith, and S. Shamai, "Compound multiple-access channels with partial cooperation," *IEEE Trans. Inf. Theory*, vol. 55, no. 6, pp. 2425–2441, Jun 2009.
- [13] H. Zivari-fard, B. Akhbari, M. Ahmadian-Attari, and M. R. Aref, "Compound multiple access channel with confidential messages," *accepted at IEEE International Conference on Communications (ICC) 2014*, Sydney, Australia, Jun 2014, available at [arxiv.org](http://arxiv.org).
- [14] A. El Gamal and Y. H-Kim, *Network information theory*, 1st ed. Cambridge, U.K: Cambridge University Press, 2012.